Exam 1 review. CSI 30. Spring 2024. Prof. Pineiro

1. Write the propositions using simpler propositions and logical connectives and determine, if possible, true or false.
   (a) If Philadelphia is the capital of the US then $7 < 3$
   (b) If you do not like to come to school then the number 4 is even.
   (c) The water is solid if and only if $4 < 2$.
   (d) 5 is a prime number or 6 is divisible by 4.

2. Let p be the proposition " $2 \leq 5$", q the proposition "8 is an even number," and r "11 is a prime number." Express the following as a statement in English and determine whether the statement is true or false:
   (a) $\neg p \wedge q$
   (b) $p \rightarrow q$
   (c) $(p \wedge q) \rightarrow r$
   (d) $p \rightarrow (q \vee (\neg r))$
   (e) $(\neg q) \rightarrow (\neg p))$

3. Translate into a logical expression: "You cannot fly on a plane if you are under 10 unless you come with your parents."

4. Show that the compound proposition

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

   is a tautology using a truth table.

5. Show that the compound proposition

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

   is a tautology **without using** a truth table.

6. Show that the compound proposition

$$(p \wedge (p \rightarrow q)) \rightarrow q$$

   is a tautology **without using** a truth table.

7. Show that the argument with premises:

$$(p \wedge t) \rightarrow (r \vee s), \quad q \rightarrow (u \wedge t), \quad u \rightarrow p, \quad \neg s, \quad q$$

   and conclusion $q \rightarrow r$ is a valid argument.

8. Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

9. Use De Morgan's Laws to find **the negation** of each of the following statements.

   (a) Pau is doing this semester Calculus I or Calculus II.

   (b) Pau is doing this semester Calculus I and Calculus II..

   (c) Every person has a subject that she/he enjoys.

10. Express each of the following statements using predicates, quantifiers, logical connectives, and mathematical operators. Domain: all real numbers.
    (a) 'There exists a real number such that if any real number is multiplied by it, we get 0'.

    (b) 'For every real number, there is a number above it'.

    (c) 'For every real number, there is a number below it'.

11. Let $D(x, y)$ be the statement "The number x divides the number y". Express, using quantifiers and logical connectives, the expression

$$P(x): \quad \text{"The number } x \text{ is a prime number"}.$$

12. Rewrite the statement

$$\neg\exists y\,(\forall x \exists z T(x, y, z) \vee \exists z U(x, y, z))$$

so that negations appear only within predicates.

13. Let $I(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet", where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.
    (a) There are two students in your class who have not chatted with each other over the Internet.
    (b) There are two students in your class who have internet connection and not chatted with each other over the Internet.
    (c) There is a student in your class who has chatted with everyone in your class over the Internet.
    (d) If a student in the class has chatted with someone else then this student has an internet connection.

14. Write the converse, inverse and the contrapositive of the statement "If you are a Computer Science major, then you know Discrete Mathematics."

15. Here are three premises:
    1. Every bird has two feet.

2. Every insect has six feet.
3. Polly has two feet.

If we conclude "Polly is a bird," have we made a valid argument? If not, why not?